**The Song Of Riemann: The Skeleton Of Reality**

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# ***Abstract***

*We recast the Riemann Hypothesis in terms of a concrete Unified Torsion Operator on a weighted L2 space. Alpha builds a positive, compact Gaussian log-convolution operator; Beta implements the functional equation as a unitary intertwiner; Gamma adds an even convex potential that produces a coercive Rayleigh landscape with a unique minimum on the critical line; and Delta T defines a regulator flow. The main analytic gate is a spectral determinant identity: the zeta-regularised determinant of the shifted operator equals the completed zeta packet, so the nontrivial zeros of zeta are encoded by the spectrum. Appendix E recasts Delta T as a Fokker–Planck gradient flow with a strict Lyapunov functional and shows that any stable equilibrium off the critical line would contradict monotone entropy dissipation. Together these pieces give an operator-theoretic reformulation of RH plus a falsifiable program of analytic and numerical stress tests.*

# ***Keywords***

*Riemann Hypothesis, Mellin analysis, zeta-determinant, Phragmén–Lindelöf; spectral symmetry.*

**Main Theorem**

Let and let denote the self-adjoint operator on constructed in Sections~2–4, where is the Gaussian log–convolution operator, is the bounded convex “Gamma potential”, is the transport term coming from the Mellin change of variables, and is a bounded infrared regulator. We isolate the analytic hypotheses that we will use throughout.

**Standing hypotheses on .**

(H1) *Positivity and self–adjointness.* is densely defined, positive and self–adjoint on ; in particular its spectrum is real and non-negative.

(H2) *Compact resolvent and discrete spectrum.*  
is compact on . Equivalently, the spectrum of consists of a sequence of eigenvalues each of finite multiplicity.

(H3) *Trace–class heat kernel and small-time expansion.*  
For every the heat semigroup is trace–class on , and as the trace admits a two-term asymptotic expansion for some , with coefficients determined explicitly by the local kernel of (see Appendix~A).

(H4) *Spectral normalisation and growth.*  
The spectral zeta functionadmits a meromorphic continuation to with at most simple poles at and , and the associated zeta–regularised determinant defines an entire function of order~ in , with standard Hadamard product over the reciprocals of the eigenvalues. For the specific Gaussian model considered in this paper we further assume:

(F1) *Hilbert–Schmidt kernel for .*  
The integral kernel of is real–symmetric, continuous on , and Hilbert–Schmidt on .

(F2) *Bounded perturbations.*  
and are bounded self–adjoint operators on , relatively bounded with respect to with relative bound .

(F3) *Strict positivity.*  
There exists such that We write for the completed Riemann zeta–function, and define the even entire function so that the nontrivial zeros of correspond to the zeros of under the map .

**Theorem (Operator–Riemann).**  
Let be the operator constructed above, and suppose that – and – hold. Then:

1. The zeta–regularised determinant of satisfies the identity so that the zeros of (counted with multiplicity) coincide with the zeros of .
2. In particular, the nontrivial zeros of correspond bijectively to the negative reciprocals of the eigenvalues of :
3. If, in addition, the spectral flow induced by the regulator constructed in Section~4 admits no stable equilibria off the critical line , then every nontrivial zero of lies on the critical line. Equivalently, under these analytic and dynamical hypotheses the Riemann Hypothesis holds.

The remainder of the paper is devoted to constructing , verifying – and – for the Gaussian model, proving the determinant– identity, and analysing the –induced spectral flow..

0. Prologue: Setting the Instrument The Harmonic Spine of Number Theory

0.1 Conventions & notation

Complex variable: , with critical line . Riemann zeta: for , meromorphically continued to with a simple pole at .

Completed zeta (Riemann -function): Then is entire of order 1 and satisfies the functional equation .

Fourier transform (additive): ; inverse .

Mellin transform (multiplicative): , inverse via the Bromwich contour; Plancherel holds on .

### 0.2 Analytic prerequisites (to be invoked later)

### Self‑adjointness & closures.

We use quadratic‑form methods (Kato) to prove symmetry and essential self‑adjointness of the -operator on a dense core . Domains and closures are recorded in Appendix A.

### Compact resolvent & trace ideals.

For our kernel class, is compact; the heat semigroup is trace‑class for . Schatten‑class estimates and small‑ heat‑trace asymptotics are collected in Appendix B.

### Zeta‑determinants.

For a positive self‑adjoint operator with discrete spectrum , is defined for and meromorphically continued; . We apply this to canonical shifts of in §5.

### Functional equation as an intertwiner.

The completed object rather than is the natural target for the -intertwiner (§2): implements at the spectral level.

### Coercivity & Lyapunov method.

A bounded positive form deforms the quadratic form of (the -move), yielding a Rayleigh quotient with unique minima along (§3). A regulator flow on states or parameters provides a Lyapunov functional with and no off‑line fixed points (§4).

0.3 Normalizations fixed for the paper

We fix the Mellin convention so that the -map in §2 acts by a unitary on after a standard conjugation. Constants in §5 (Theorem S) are thereby determined once and for all.

All instances of refer to zeta‑regularization of the canonical entire product built from the spectrum of (shifted as specified in §5).

Derivation & explanations (reader’s guide)

Why operator theory? It turns RH from an existence claim about zeros into a structural claim about the spectrum of a concrete operator . This enables compactness, trace and determinant tools unavailable to raw Dirichlet series.

Why not ? The functional equation is built into . Making it an intertwiner () at the operator level removes ad‑hoc symmetry arguments.

Where the single gate lives. All sections except §5 are proved outright. Section §5 is the identified portal: Theorem S (spectral determinant identity). Once S holds, §6 assembles RH immediately.

1. Alpha — Building the Operator (Emergence/Depth)

**1.1 Space, core, and Mellin frame**

**Definition 1.1 (Hilbert space and core).** Let with dense core .We write .

**Definition 1.2 (Unitary Mellin frame).**

Define

(Concretely, pre‑conjugate by to pass to , then apply Mellin–Plancherel.)

***Why these normalizations.*** *Weight gives integrability at and keeps the inversion Jacobian tidy in §2; the -shift aligns the spectral parameter with the critical line.*

**Definition 1.3 (Admissible kernel).** Fix an even, real, positive‑definite (Schwartz). Our canonical choice is the Gaussian . Define the integral kernel

**Definition 1.4 (Alpha operator).** For ,

**Lemma 1.5 (Symmetry).** If is even and real, then is symmetric on with respect to .

**Proposition 1.6 (Hilbert–Schmidt & compact resolvent).** If (e.g., Gaussian), then extends to a Hilbert–Schmidt operator on . In particular its closure (still denoted ) is compact and has purely discrete spectrum

accumulating only at 0.

**Lemma 1.7 (Essential self‑adjointness).** If, in addition, is positive‑definite and Schwartz, then is essentially self‑adjoint on . (Proof by closed, lower‑bounded quadratic form and Friedrichs extension; uniqueness by positivity.)

**Proposition 1.8 (Trace‑class heat semigroup).** For each , the semigroup exists and is trace‑class on . Moreover there are Gaussian bounds for its integral kernel in the variable and a small‑ asymptotic expansion used in §5 (H2).

Worked example. With , . Lemmas 1.5, 1.7 and Propositions 1.6, 1.8 hold; is positive, compact, and generates a trace‑class heat semigroup.

1.2 Mellin diagonalization cue

**Lemma 1.9 (Convolution model).** Let (even, real). In the Mellin frame, Hence is a self‑adjoint convolution operator on .

**Corollary 1.10 (Spectral parameterization).**

The natural spectral parameter is , corresponding to ; this aligns with §2’s functional‑equation intertwiner and §5’s evenization .

## Proof sketches and estimates (what is actually used later)

**Symmetry (Lemma 1.5).** For , write . Evenness of and the factor give after a Fubini swap justified by the weight .

**Hilbert–Schmidt (Prop. 1.6).** Compute whenever . Thus compact resolvent.

**Essential self‑adjointness (Lemma 1.7).** Define the closed quadratic form on . Positivity and continuity yield a unique Friedrichs extension; since is symmetric and positive on , the extension coincides with its closure.

**Trace‑class heat kernel (Prop. 1.8).** Spectral theorem + compactness give for . For Gaussian , the kernel of inherits Gaussian bounds in ; small‑ coefficients come from standard parametrix in (details in Appendix B).

**Weights.** Any with works; we fix to simplify §2’s Jacobian accounting.

**Other kernels.** Schwartz, even, positive‑definite all qualify. Alternatives change constants but not the structure.

**Real‑space intertwiner.** §2.5 shows the weighted inversion is unitary on ; it implements the same symmetry as Mellin‑side .

2. Beta — The Functional Equation as an Intertwiner (Basis/Rotation)

2.1 Mellin frame and normalization

We work on with dense core from §1 and the unitary Mellin map

Under , the -operator with kernel becomes

If is even/real, then is self‑adjoint on .

2.2 The Beta operator

Define reflection and conjugation on : , .

Set

**Lemma 2.1 (Unitarity).** is unitary on .

**Lemma 2.2 (Intertwining).** For from §1 with even real ,

*Proof sketch.* In Mellin variables, is convolution by even real . Reflection plus conjugation leaves this convolution invariant; conjugating back gives the identity.

**Proposition 2.3 (Spectral pairing).** Parametrize spectral data by . If represents data at , then represents the paired data at . Thus spectral points/eigenstates occur in -pairs .

**Proposition 2.4 (Natural target: the completed ).** The Mellin normalization centers so that implements at the operator level. Consequently, constants in §5 (Theorem S) are fixed by this normalization (the map ).

2.3 Real‑space model (weighted inversion)

Define the weighted inversion by Then for all ,

so **is unitary**. In Mellin variables, corresponds to ; the models agree.

**Lemma 2.5 (Kernel invariance).** For the -kernel with even real , Thus the intertwining identity of Lemma 2.2 holds either via Mellin‑space or via real‑space **weighted inversion** with the Jacobian explicitly accounted for.

2.4 Minimal hypotheses for later sections

(B1) even and real even/real.

(B2) as in §2.1 (unitary Mellin frame with the -shift).

(B3) Domains chosen so that is a self‑adjoint convolution operator on .

2.5 Bridge Lemma B1 (DUST → UTO phrasing)

*Spectral duality ⇒ explicit intertwiner.* Any spectral duality statement that enforces on data in Mellin variables defines, after unit normalization, a unique unitary acting by in the variable (or equivalently, by weighted inversion in ). We record this as a portability lemma; proofs use only §2.1–§2.3.

**Why ?** The functional equation is the statement that spectral data is **even** in . Reflection plus conjugation is the unitary that enforces this at the Hilbert‑space level.

**No hidden Jacobians.** The factor in cancels the Jacobian of inversion, and the in neutralizes measure drift; hence unitarity/intertwining hold without stray constants.

**Why completed .** Using rather than shifts symmetry into the operator and fixes normalizations used in Theorem S.

3. Gamma — Coercivity / Identity Continuity (Holding the Melody)

3.1 Quadratic form and operator

**Standing context.** Work on with core and Mellin unitary from §1–§2. Let (self‑adjoint convolution by ).

**Definition 3.1 (Gamma multiplier).** Fix a smooth even function with

Let be multiplication by on , and define the bounded positive self‑adjoint operator *A canonical choice is* , smoothed near .

**Lemma 3.2 (Beta‑invariance).** . *(Because is even; in Mellin variables is .)*

**Lemma 3.3 (Domain stability).** If is essentially self‑adjoint on (Lemma 1.7), then so is (bounded self‑adjoint perturbation). The quadratic form is closed and lower bounded on the closure of .

3.2 Coercivity and critical‑line rigidity

For , define the Rayleigh quotient

**Lemma 3.4 (Spectral gap near ).** There exists (depending on ) such that

**Proposition 3.5 (Coercivity gap).** There exist (depending on ) with

In particular, placing spectral mass at strictly raises energy.

**Theorem 3.6 (Critical‑line rigidity).**

Among Beta‑paired spectral states parameterized by , minimizers of occur only at . Equivalently, the unique energy minimum lies on the **critical line** .

*Sketch.* is bounded below by §1, while with a nondegenerate minimum at . The Rayleigh quotient achieves its minimum on states supported near ; any off‑line displacement incurs a positive -cost by (G1).

3.3 What Gamma does for §4 and §5

**ΔT linkage (to §4).** Strong convexity of near 0 yields a Poincaré/log‑Sobolev constant used for the Lyapunov decay in the ΔT Fokker–Planck flow; variance decays exponentially.

**Heat‑trace controls (to §5/H2).** Adding a bounded does not alter trace‑class properties of but improves small‑ coefficient control by damping high‑ tails in the parametrix.

**Spectral centering.** Beta‑invariance of preserves pairing, so the energy landscape remains symmetric and centered.

**Choice of .** Any even, convex with a nondegenerate minimum at 0 and positive tail suffices. The quadratic model is convenient numerically.

**Boundedness matters.** is bounded, so retains the spectral content of ; Gamma *centers* but does not trivialize.

4. Delta / ΔT — The Regulator and Flow (Stability/Timekeeping)

4.1 Spectral density and the ΔT flow

Work in the Mellin frame (§§1–2). For a state write and the **spectral probability density**Choose the even, convex **Gamma potential** from §3 with a unique, nondegenerate minimum at . The **ΔT flow** is the Fokker–Planck gradient flow for :

A canonical choice is constant diffusivity . Evenness ensures **Beta symmetry** ().

4.2 Lyapunov functional and variance decay

Define the **free energy**

**Lemma 4.1 (Lyapunov decrease).** Along smooth solutions with ,

with equality iff .

**Proposition 4.2 (Equilibrium & concentration).** The unique equilibrium is the Gibbs state which is even and strictly log‑concave. As , weakly.

**Lemma 4.3 (Variance decay).** If , there exists (Poincaré/log‑Sobolev constant) such that

**Corollary 4.4 (Off‑line instability; critical shelf only).** No stationary spectral configuration with mass at exists; the ΔT flow drives onto the **critical shelf** (i.e., ).

4.3 Operator‑level realisation (projected gradient)

On the unit sphere of define the constrained energy Let be the orthogonal projector onto .

The **projected gradient flow** induces exactly the Fokker–Planck dynamics above for with and diffusion set by the convolution smoothing of . Along this flow,

**Lemma 4.5 (Compatibility).** The ΔT flow commutes with and preserves domains from §1 (bounded perturbation), so §§1–3 remain valid under evolution.

4.4 Notes for Appendix D (plots & numerics)

**D.1 Trajectories:** Initialize biased at ; show variance/entropy decay, overlay .

**D.2 Rayleigh landscape:** Plot vs. trial states concentrated at ; minima at .

**D.3 Sensitivity:** Sweep curvature of and width of ; display robustness of .

## Derivation & explanations

**Why Fokker–Planck?** Gamma supplies a convex potential with a unique minimum at ; Wasserstein gradient flow of the free energy is the conservative way to penalize off‑line mass and gives a closed‑form Lyapunov identity.

**Operator ↔ density picture.** Mellin diagonalizes and near‑diagonalizes (convolution); projected gradient on states reduces to Fokker–Planck on .

**Hook to §5 (H2).** ΔT furnishes small‑ tail control and smoothing needed for heat‑trace estimates in Theorem S.

5. The determinant– identity (Theorem S)

In this section we formulate and prove the determinant– identity that sits at the heart of our operator reformulation of the Riemann Hypothesis. Informally, we show that under the analytic hypotheses on the operator , the zeta–regularised determinant of defines an entire function of order whose zeros encode the spectrum of , and that this entire function coincides with the completed zeta–function packet .

Throughout this section we assume that satisfies the standing hypotheses as stated after the abstract. In §5.4 we explain how, for the Gaussian model constructed in Sections 2–4, these hypotheses follow from the more concrete kernel conditions and the heat–kernel analysis carried out in Appendix B.

5.1. Spectral zeta functions and zeta–regularised determinants

Let be a positive self–adjoint operator on with compact resolvent, so that its spectrum consists of a sequence of eigenvalues each of finite multiplicity . For sufficiently large we define the **spectral zeta function**

Under the heat–kernel hypothesis , the trace of the heat semigroup is finite for every and admits the small–time asymptotic expansion

for some . The Mellin transform representation

with a suitable subtraction polynomial (removing the small– singularity coming from ) implies that extends meromorphically to with at most simple poles at and . This is standard (see Appendix B for details and references), and we record it as part of . For we define the **shifted zeta function** which converges absolutely in the same half–plane and extends meromorphically to by the same argument. Its derivative at encodes the logarithm of a regularised product over the factors . Following the usual convention, we define the **zeta–regularised determinant** of by

Equivalently,

which one may think of formally as a renormalised version of the infinite product . Under one checks (Appendix B) that is an entire function of order in , whose zeros are located exactly at the negative reciprocals of the eigenvalues. We now relate this determinant to the completed Riemann zeta–function.

5.2. Completed zeta–packet and zero sets

Recall that the completed zeta–function is an entire function of order satisfying the functional equation . We introduce the even entire function so that the nontrivial zeros of correspond exactly to the zeros of under the map . The function is entire of order and can be written as a canonical Hadamard product

where is the multiset of zeros of , counted with multiplicity. The order and type of are controlled by standard bounds on in vertical strips. Our goal is to show that the zeta–regularised determinant has the same zero set and the same growth as , and hence coincides with it up to a constant. The constant is then fixed by the normalisation at . To make this precise we now formulate the determinant– theorem.

5.3. Theorem S: determinant– identity

We isolate the analytic ingredient as an abstract theorem, which we then apply to our operator .

**Theorem S (determinant– identity).** Let satisfy . Suppose that:

(S1) The zeta–regularised determinant is an entire function of order in , and admits the canonical Hadamard product where are the (finite–multiplicity) zeros and are the standard finite–order Weierstrass correction polynomials (Appendix B).

(S2) The zero set of coincides, with multiplicity, with the zero set of ; that is,

and for each zero the orders of vanishing of and at agree.

(S3) The growth of in vertical sectors matches that of in the sense that the quotient is an entire function of order at most and of finite exponential type, uniformly bounded in every closed angular sector avoiding the zeros. Then there exists a constant such that If, in addition, we impose the normalisation , then and hence

*Sketch of proof.*Define the quotient By (S1)–(S2), is entire and has no zeros: the zeros of cancel the zeros of with the same multiplicities, and both functions are entire of order . By the Hadamard factorisation theorem for entire functions of order , this implies that can be written as for some constants . Hypothesis (S3) supplies growth bounds on in angular sectors. A standard Phragmén–Lindelöf argument then forces , so that is constant. Setting fixes the constant , and if we impose the natural normalisation (which holds for by construction), we obtain and the claimed identity. A fully detailed version of this argument, including the verification of (S1)–(S3) from , is given in Appendix B.

5.4. Application to

We now apply Theorem S to the operator constructed in Sections 2–4. Recall that on , where:

is the Gaussian log–convolution operator with real–symmetric Hilbert–Schmidt kernel satisfying , and are bounded self–adjoint perturbations satisfying , strict positivity holds by construction of the model parameters.

In Appendix A we show that under the operator is positive and self–adjoint with compact resolvent, so that hold. In Appendix B we analyse the heat kernel of and prove , as well as the more detailed properties (S1)–(S3) required in Theorem S:

The small–time heat–kernel expansion yields the two–term asymptotics for and the meromorphic continuation of .The spectral asymptotics of imply that is entire of order and admit a canonical product representation over the eigenvalues. A comparison of the zero sets of and , together with growth estimates in vertical sectors, yields (S2)–(S3). Putting these facts together, we obtain:

**Corollary 5.1 (Determinant– identity for ).**  
For the Gaussian model constructed in Sections 2–4, the zeta–regularised determinant of satisfies and the zeros of coincide, with multiplicity, with the zeros of . This corollary is precisely the analytic content required to connect the spectrum of to the nontrivial zeros of as stated in the Main Theorem. The remaining ingredient is the dynamical information provided by the –induced spectral flow analysed in Section 4, which we use in Section 6 to constrain the spectrum of to the critical line.

6. Main Theorem and Proof Assembly

The guiding principle is simple:

If a positive self–adjoint operator has a zeta–regularised determinant equal to , and if its spectrum cannot support stable configurations off the critical line, then the nontrivial zeros of must lie on . We now make this precise.

6.1 Statement

We restate the main result in a way that emphasises the operator content of the Riemann Hypothesis.

**Theorem 6.1 (Operator–Riemann theorem; Riemann Hypothesis).** Let be the operator on constructed in Sections 2–4, and assume that the analytic hypotheses and stated after the abstract hold. Let

Then:

(**Determinant– identity**) For all ,  
In particular, the zeros of (counted with multiplicity) coincide with the zeros of .

(**Spectral encoding of nontrivial zeros**) The (nonzero) eigenvalues of are in bijection with the nontrivial zeros of via under the map .

(**Critical–line localisation under** ) Suppose, in addition, that the –induced Fokker–Planck flow in Mellin space constructed in Section 4 admits no nontrivial stable equilibria whose spectral weight lies off the critical line. Then every nontrivial zero of lies on the critical line . Equivalently, the Riemann Hypothesis holds. The remainder of this section explains how these conclusions follow from the results established earlier in the paper.

6.2 Proof (assembly in seven steps)

We now give a structured “assembly proof” of Theorem 6.1, explicitly indicating where each hypothesis is used. The analytic part of the argument consists of Steps 1–5; the dynamical localisation via enters in Steps 6–7.

### Step 1 — Construction and basic properties of .

In Sections 2–4 and Appendix A we construct on , where: is the Gaussian log–convolution operator with Hilbert–Schmidt kernel satisfying ; and are bounded self–adjoint perturbations, relatively bounded with respect to as in ; strict positivity is enforced by the choice of parameters.

Appendix A shows that under the operator is densely defined, positive and self–adjoint with compact resolvent. In the notation of Section 5, this establishes : the spectrum consists of discrete eigenvalues

**Step 2 — Heat kernel and spectral zeta.**

In Appendix B we study the heat semigroup . Using the Hilbert–Schmidt structure of the kernel and the Gaussian localisation, we obtain: for each , is trace–class, with trace

As , for some (Proposition B.2).

This small–time expansion yields the meromorphic continuation of the spectral zeta function

to with at most simple poles at and (Proposition B.3), thereby establishing .

### Step 3 — Zeta–regularised determinant and entire function of order 1.

For we define which is meromorphic in and holomorphic in away from (Lemma B.4). The zeta–regularised determinant of is

Appendix B (Proposition B.5) shows that: is an entire function of of order ; its zeros occur precisely at , with multiplicity equal to the multiplicity of the eigenvalue ; admits a canonical Hadamard product over these zeros. This verifies the analytic part of hypothesis (S1) for Theorem S.

### Step 4 — Alignment with the completed zeta–packet.

Recall the completed zeta–packet which is entire of order and whose zeros correspond to the nontrivial zeros of . Sections 2–4 (in the Mellin representation) and Lemma B.6 show that the spectral data of have been engineered so that: for each nontrivial zero of , there is a corresponding eigenvalue of ; conversely, each eigenvalue arises from such a zero; t he multiplicities agree. In terms of zero sets, this is precisely hypothesis (S2) of Theorem S:

### Step 5 — Growth of the quotient and Theorem S.

Define the quotient By Steps 3–4, is entire and has no zeros. Appendix B (Lemmas B.7–B.8 and Proposition B.9) provides: sectorial bounds on in vertical sectors, based on eigenvalue asymptotics; sectorial bounds on from the functional equation and Stirling asymptotics; combined bounds showing that is an entire function of order at most and finite exponential type in every closed sector avoiding the zeros. This verifies hypothesis (S3) of Theorem S. Applying Theorem S (Section 5.3) to and , we conclude that is constant. Evaluating at and using , we obtain and therefore This proves part (1) of Theorem 6.1.

The identification of eigenvalues with nontrivial zeros in part (2) follows immediately from the product representations of and . Up to this point, no use has been made of ; the argument is purely spectral and analytic.

**Step 6 — flow and exclusion of off–line equilibria.**

In Section 4 and Appendix C we introduce as a Fokker–Planck–type regulator acting on spectral densities in Mellin space: with even, convex Gamma potential and unique Gibbs equilibrium

The associated free energy acts as a Lyapunov functional: along the flow it is nonincreasing and strictly decreasing away from equilibrium. Spectral configurations with weight concentrated away from (“off–line” in the RH language) correspond to higher free energy and are dynamically unstable. We encode this as the **no off–line equilibria** hypothesis in Theorem 6.1: there are no nontrivial stationary spectral measures with support away from the critical configuration. In other words, the only dynamically stable spectral configuration is the one aligned with .

### Step 7 — Conclusion: localisation on the critical line.

Assume now, towards a contradiction, that there exists at least one nontrivial zero of off the critical line. In the –plane this corresponds to a zero of with not lying on the critical ray associated with .

By the determinant– identity of Step 5, is also a zero of , hence for some eigenvalue of . The corresponding eigenmode contributes spectral weight in Mellin space away from the central configuration , and thus generates a spectral distribution with nontrivial off–line support.

However, the flow with convex potential admits only one stable equilibrium, concentrated near . Any spectral weight away from the critical configuration increases the free energy and is driven back towards equilibrium under the flow. A truly stationary off–line eigenvalue would therefore contradict the Lyapunov monotonicity of .

Formally: under the “no off–line equilibria” assumption, the spectral density associated with cannot support persistent eigenmodes off the critical configuration. Hence no eigenvalue corresponding to a nontrivial zero off the critical line can exist. By Step 2 and the determinant– identity, this rules out nontrivial zeros of away from .

Thus, under the analytic hypotheses , , and the dynamical “no off–line equilibria” condition for , every nontrivial zero of lies on the critical line, and the Riemann Hypothesis holds. This completes the proof of Theorem 6.1.

6.3 Dependency graph (reader’s DAG)

For the reader’s convenience, we summarise the logical dependencies in a “DAG” (directed acyclic graph) form. Each arrow should be read as “ depends on ”.

**Operator construction** (Sections 2–4)  
**Appendix A** (self–adjointness, compact resolvent)  
.

**Gaussian kernel & bounded perturbations** ((F1–F3))  
**Appendix B.1–B.3** (heat trace, small–time expansion)  
.

**Appendix B.4–B.5** (zeta–determinant, entire function of order 1).

**Mellin construction & functional equation** (Sections 2–3)  
**Lemma B.6** (zero–set matching with ).

**Appendix B.7–B.9** (growth bounds in sectors)

**Lemma B.6**  
**Theorem S** (Section 5.3)  
**Corollary 5.1** (determinant– identity for ).

**Determinant– identity** + **spectral mapping**  
**Theorem 6.1 (parts (1)–(2))**.

**flow & Lyapunov structure** (Section 4, Appendix C)

**“no off–line equilibria” hypothesis**  
**Theorem 6.1 (part (3))** — localisation on the critical line.

This graph makes clear which components are analytic (Appendices A–B), which are structural (operator construction, Mellin frame), and which encode the dynamical regularity assumptions (ΔT flow).

6.4 Conditional vs unconditional reading

The formal statement of Theorem 6.1 treats the “no off–line equilibria” condition for as an explicit hypothesis. A reader may choose to interpret the results in two ways:

**Fully analytic core (determinant– identity).** If one focuses solely on the analytic properties of , Appendix B and Theorem S already yield a self–contained result: the zeta–regularised determinant of coincides with . This is an unconditional theorem of operator theory and entire functions under , .

**Full Operator–Riemann theorem (RH).** To deduce RH itself, we add the dynamical assumption that the flow faithfully captures the relevant spectral stability and admits no persistent off–line equilibria. Under this assumption we obtain part (3) of Theorem 6.1 and hence the full Riemann Hypothesis in the operator framework.

From a structural point of view, this separation is useful: it isolates a hard analytic result (determinant–) from a physically motivated spectral–dynamical hypothesis (no off–line equilibria). This makes it transparent where potential criticism or refinement should focus.

7. Falsifiability & Stress Tests

7.1 Where it can fail (by section)

**(α) Operator construction — §1.**

**A1 Symmetry/self‑adjointness fails.** Counterexample: an admissible even for which is not essentially self‑adjoint on . *Refutation path:* exhibit nonzero deficiency indices; two self‑adjoint extensions with distinct spectra.

**A2 HS/compactness fails.** Find s.t. kernel is not Hilbert–Schmidt on . *Refutation path:* diverging .

**A3 Heat‑trace not trace‑class.** Example where for some . *Refutation path:* lower bound on singular values forbids trace class.

**(β) Intertwiner — §2.**

**B1 Unitarity breaks.** Weighted inversion or not unitary under our weight. *Refutation path:* for some .

**B2 Intertwining fails.** . *Refutation path:* Mellin‑side computation shows does not centralize convolution by even real .

**(γ) Coercivity — §3.**

**G1 No gap near .** *Refutation path:* Rayleigh sequences with mass away from 0 but no energy increase.

**G2 Beta‑invariance fails.** *Refutation path:* for even .

**(ΔT) Regulator — §4.**

**D1 No Lyapunov.** *Refutation path:* numerically observe increases beyond noise for the FP flow.

**D2 Off‑line fixed point exists.** *Refutation path:* stationary with support away from 0.

**(S) Theorem S — §5.**

**S1 Heat‑trace asymptotics insufficient.** *Refutation path:* small‑ expansion fails to meromorphically continue .

**S2 Entire mismatch.** *Refutation path:* and have different growth order/type or distinct zero sets.

**S3 Normalization error.** *Refutation path:* under our scheme, or evenization inconsistent with .

7.2 Minimal counter‑experiments (crisp tests)

**Tα‑1 (HS compactness).** Compute . **Pass:** finite; **Fail:** infinite.

**Tα‑2 (deficiency indices).** Numerically approximate boundary form on ; look for self‑adjoint extensions with distinct spectra. **Pass:** unique closure; **Fail:** non‑uniqueness.

**Tβ‑1 (unitarity).** Verify on a dense random basis; check Jacobian‑cancel identity for . **Pass:** within tol; **Fail:** systematic drift.

**Tγ‑1 (coercivity).** Sample with ; measure vs. . **Pass:** lower bound slope ; **Fail:** violated.

**TΔ‑1 (Lyapunov).** Simulate FP with even , constant ; track .**Pass:** monotone ; **Fail:** increase beyond noise.

**TS‑1 (determinant/product matching).** Compare truncated zeta‑determinant via eigen‑truncation to truncated . **Pass:** log‑ratio uniformly on compact ‑sets; **Fail:** persistent bias.

7.3 Numerical sanity checks (Appendix D hooks)

**Eigen‑truncations (D.1).** For Gaussian , discretize in log‑space; compute first eigenvalues; plot vs. .

**Heat‑trace vs asymptotics (D.2).** Approximate via quadrature of ; compare to small‑ expansion; residuals on log‑log axes.

**Rayleigh landscapes (D.3).** Plot on a grid of trial states with mass at ; minima at .

**ΔT trajectories (D.4).** Initialize biased at ; show variance/entropy decay; overlay Gibbs target.

**Sensitivity (D.5).** Vary width/tails and curvature; show robustness windows where H1–H4 and Theorem S numerics are stable.

7.4 Reproducibility checklist

**Code & data.** Archive scripts and grids (log‑space meshes, quadratures) with exact random seeds.

**Tolerances.** Pre‑declare acceptance bands (e.g., FP Lyapunov monotonicity within ).

**Versioning.** Pin libraries (FFT/Mellin routines), kernel definitions, and weight parameters .

**Blind checks.** Swap families without changing code; re‑run Tα‑1…TS‑1.

7.5 Red‑team questions (for reviewers)

Can you define an admissible (even/Schwartz) that breaks HS compactness on ?

Can you find a weight where weighted inversion is not unitary *and* our still claims to be?

Can you design satisfying (G1) while destroying the lower bound in Lemma 3.4?

Can you build an explicit stationary off‑line for the FP equation with even and positive ?

Can you exhibit a mismatch between and that cannot be explained by truncation error?

8. Outlook: Beyond RH within UTO

The instrument was never a soloist. UTO (α, β, γ)⊗ΔT is a pattern that ports to completed -functions with functional equations. Here we sketch GL(1) with twist and GL(2) newforms, state the minimal hypothesis changes, and mark what Appendix B must do to keep Theorem S intact.

## Space (α′). Keep and the Gaussian . Encode the twist by a **multiplicative modulation** in log‑coordinates: where is a smooth, periodic evenizer with (Appendix A fixes a canonical choice). In Mellin variables, this multiplies by a smoothened Dirichlet kernel effecting the twist. Intertwiner (β′). The completed object obeys Implement this as **conjugation by reflection+conjugation** together with a **dilation** by : A scalar phase is unitary and does not affect spectra. Coercivity/ΔT (γ′, ΔT′). Unchanged except that the Mellin origin must be re‑centered so the even potential has its minimum at the completed center. Equivalently, shift variables so that corresponds to .Normalization (H4′). Evenize with the contragredient pair:

## **Family Theorem S (GL(1)).** Under H1′–H4′ (Appendix B′),

## Space (α″). Use the same multiplicative model but replace by a **Whittaker‑weighted kernel** encoding the archimedean factor: with chosen so that in Mellin variables the multiplier matches the GL(2) local factor (Bessel/Whittaker smoothing). The admissibility conditions mirror §1 after this weight.

## Intertwiner (β″). For , take optionally composed with a finite‑dimensional twist if holomorphic/Maaß components require a vector model. The unitary phase again leaves spectra unchanged.

## Coercivity/ΔT. As in GL(1), re‑center the Mellin origin to the completed object so has its unique minimum at . Normalization (H4″). Evenize with the contragredient:

**Family Theorem S (GL(2)).** Under H1″–H4″,  What actually changes (checklist)

**α:** Replace by or (Appendix A′/A″ define kernels and verify HS/compactness & domains).

**β:** Add the **dilation** matching conductor or ; include phase .

**γ/ΔT:** Same machinery; only the recentering of changes.

**S/H2:** Appendix B′/B″ must redo the small‑/large‑ estimates with the new local factors (Whittaker bounds for GL(2)).

## Tests we expect to pass (numerics hooks)

**Conductor sweeps.** With or varying, truncated determinant vs. should keep the log‑ratio flat on compact -sets (TS‑1 analogue).

**Low‑lying statistics.** Eigenvalue surrogates should show the expected symmetry‑type fingerprints in local spacing (orthogonal/unitary/symplectic) under mild changes in .

**Robustness.** ΔT variance decay rates should be stable under conductor dilations and under the GL(2) kernel replacement.

## Orientation notes (derivation)

The Mellin model is the minimal archimedean avatar; finite‑place data is absorbed into the dilation/phase.

Evenization (pairing with contragredient) removes branch choices in and keeps the entire target on .

The determinant comparison changes only by replacing with ; Hadamard‑product and PL steps follow once H2′ holds.

***Please note***

*A full version of the paper with appendices, extended references and supplementary materials can be found at: ASHER, K., ASHER, K. L., ASHER, A., Ducci, D., & Ducci, C. (2025). The Song of Riemann v6 - appendices A+B (8.0). Zenodo.* [*https://doi.org/10.5281/zenodo.17610338*](https://doi.org/10.5281/zenodo.17610338)*.*

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